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THEORETICAL TOOLS
FOR A FUTURE e^+e^- LINEAR COLLIDER[1]

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INTRODUCTION

Precision measurements at LEP, SLC, and the Tevatron rendered the last decade the era of high-precision physics. A future e^+e^- linear collider (LC), such as TESLA [1], the NLC [2], or the GLC (former JLC) [3], does not only offer an even greater physics potential, but in turn represents a great challenge for theorists to understand phenomena at the experimentally achievable level of precision.

For instance, returning again to the Z-boson resonance in the "GigaZ" mode of TESLA (where about 10^9 Z bosons can be produced within 50–100 days of running) allows for a repetition of the LEP1/SLC physics program with roughly an order of magnitude higher precision (see also Ref. [4]). Specifically, the uncertainty in the effective weak mixing angle could be reduced from 1.7×10^{-4} to 1.3×10^{-5}. For a theoretical description of the Z resonance at this level of accuracy full two-loop calculations of the observables as well as the knowledge of leading

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higher-order effects are clearly necessary. A scan over the W-pair production threshold could provide a sensitivity to the W-boson mass of about $7\,\mathrm{MeV}$, which should be compared with the present error of $34\,\mathrm{MeV}$, resulting from the kinematical reconstruction of W bosons. The present approach of approximating the radiative corrections to $e^+e^- \to WW \to 4\,\mathrm{fermions}$ by an expansion about the double resonance is not applicable (or at least not reliable) in the threshold region where singly- or non-resonant contributions become important. The only solution seems to be the full treatment of the complete four-fermion production processes at the one-loop level, including higher-order improvements.

At energies exceeding the reach of LEP2, many new processes will be accessible, such as top-quark pair production, Higgs production (if the Higgs boson exists), or reactions with new-physics particles, as e.g. predicted by SUSY models. Most of these heavy particles are unstable, so that their production eventually leads to many-particle final states. For example, the production of $t\bar{t}$ pairs or of a Higgs-boson with an intermediate or large mass ($M_H > 2M_W$) leads to six-fermion final states. To exploit the potential of a LC, predictions for such reactions should be based on full transition matrix elements and improved by radiative corrections as much as possible. The higher level of accuracy at a future LC does, however, not only call for proper event generators for many-particle final states. "True" event generators, i.e. including parton showering and hadronization, have to be improved as well.

In this brief article the main progress on precision calculations and event generators that has been achieved in the "Loopverein" and "Generators" working groups of the Extended ECFA/DESY Study since the appearance of the TESLA TDR [1] is reviewed. More studies on the physics potential of a LC in view of electroweak and strong interactions, top-quark physics, Higgs physics, and new physics searches, in particular supersymmetry, are summarized in Ref. [5].

HIGH-PRECISION OBSERVABLES AND MULTI-LOOP CALCULATIONS

Precision calculations for μ decay

The precision measurement of the muon lifetime, or equivalently of the Fermi constant G_μ, sets an important constraint on the SM parameters,

$$G_\mu = \frac{\pi\alpha(0)}{\sqrt{2}M_W^2 s_w^2}\,(1 + \Delta r), \qquad (1)$$

where $s_w^2 = 1 - c_w^2 = 1 - M_W^2/M_Z^2$ and the quantity Δr comprises the radiative corrections to muon decay (apart from the photonic corrections in the Fermi model). In the past it has become common practice to implicitly solve this relation for the W-boson mass M_W, thus yielding a precise prediction for M_W that can be compared with the directly measured value. Recently the full prediction at the two-loop level has been completed. In detail, first the contributions from closed fermion loops and from bosonic loops involving Higgs-boson exchange were calculated in Ref. [6] by making use of the FEYNARTS package [7] and the program TWOCALC [8] both written in MATHEMATICA. The algebraic reduction leads to two-loop master integrals which are evaluated by semi-analytical methods. The full bosonic corrections have been calculated in Refs. [9] and [10]; Ref. [9] includes also a recalculation of the fermion-loop correction. In the former calculation the diagrams were generated with the C++ library DIAGEN (by Czakon) and evaluated using semi-analytical methods. In the latter case the graphs were generated with the package DIANA [11] and evaluated by asymptotic expansions. The results of Refs. [6, 9, 10] are in good numerical agreement [12].[1] The two-loop fermionic corrections influence the M_W prediction at the level of $\sim 50\,\mathrm{MeV}$, where the bulk of this effect is due to universal, top-mass enhanced corrections to the ρ-parameter, which are proportional to m_t^4 or m_t^2. The non-universal two-loop fermionic corrections have an impact of up to $4\,\mathrm{MeV}$, the two-loop bosonic corrections of only $1-2\,\mathrm{MeV}$.

The predictions at the two-loop level have been further improved by universal higher-order corrections to the ρ-parameter. The corrections of $\mathcal{O}(G_\mu^2 m_t^4 \alpha_s)$ and $\mathcal{O}(G_\mu^3 m_t^6)$ have been calculated for arbitrary M_H in Ref. [13] (for other universal corrections to $\Delta\rho$ and Δr see references therein) and were found to change M_W at the level of $5\,\mathrm{MeV}$ and $0.5\,\mathrm{MeV}$, respectively. The Feynman diagrams were generated using QGRAF [14] and asymptotically expanded with the program EXP [15]; the resulting massive three-loop tadpole integrals were evaluated with MATAD [16].

Figure 1 compares the prediction for M_W, including the above-mentioned two-loop and leading three-loop effects, with the experimental value. Note that the shown parametric uncertainty is much larger than the estimated theoretical uncertainty, which is about $3-4\,\mathrm{MeV}$ [17, 18]. Comparing this estimate with the aimed precision of $7\,\mathrm{MeV}$ in the M_W determination at a future LC, the prediction of the W-boson mass from muon decay is in rather good shape.

Precision observables on the Z resonance

In order to describe the Z-boson resonance at LEP1 within satisfactory precision it was possible to parametrize the cross section near the resonance in such a way [19] that a Born-like form with generalized "effective" cou-
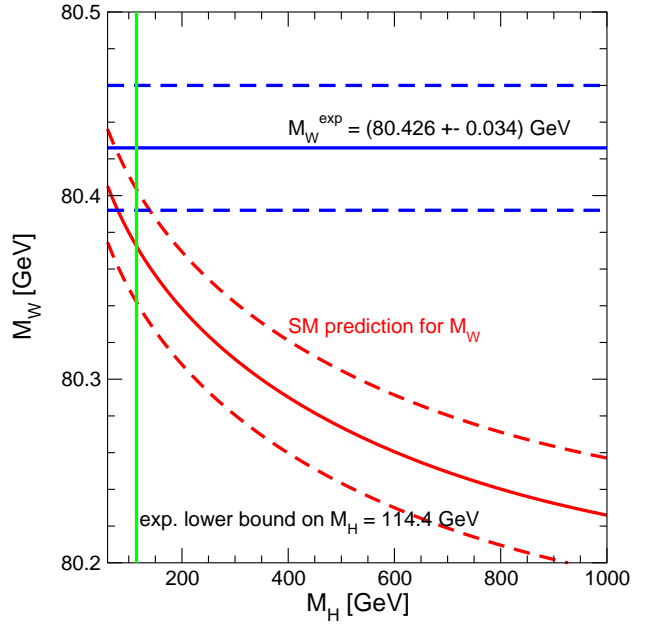

Figure 1: Theoretical prediction of the W-boson mass with the parametric error from uncertainties in the top-quark mass and the running electromagnetic coupling, in comparison with the experimental value M_W^{exp} (plot shown in Ref. [17])

plings is convoluted with QED structure functions modeling initial-state radiation (ISR). From these effective Z-boson–fermion couplings so-called "pseudo-observables" were derived, such as various asymmetries, the hadronic Z-peak cross section, partial Z-decay widths, etc. Following the formal tree-level parametrization of the couplings, an "effective weak mixing angle", usually given as $\sin^2 \theta_{f,\mathrm{eff}}$, was derived for each fermion. Among these parameters the leptonic variable $\sin^2 \theta_{\mathrm{lep,eff}}$ plays a particularly important role, since it is measured with the high accuracy of 1.7×10^{-4} and is very sensitive to the Higgs-boson mass. The state-of-the-art in the precision calculations of the pseudo-observables, which is implemented in the programs ZFITTER and TOPAZ0 (see Ref. [20] and references therein), did not change very much since the release of the TESLA TDR [1]. For instance, the estimated theoretical uncertainty in $\sin^2 \theta_{\mathrm{lep,eff}}$ is still $\sim 6 \times 10^{-5}$. A critical overview about high-precision physics at the Z pole, in particular focusing on the theoretical uncertainties, can be found in Ref. [21] (see also Ref. [4]).

Whether the pseudo-observable approach will also be sufficient for Z-boson physics at the high-luminosity GigaZ option remains to be investigated carefully. In any case, tremendous theoretical progress will be needed to match the aimed GigaZ precision on the theoretical side. For example, the expected experimental accuracy in $\sin^2 \theta_{\mathrm{lep,eff}}$ is about 1.3×10^{-5}, i.e. about a factor 4 below the present theoretical uncertainty. A full control of observables at the two-loop level, improved by leading higher-order effects, seems to be indispensable.

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[1] The results of Ref. [6] for M_W have been corrected at the level of $\sim 1\,\mathrm{MeV}$ recently.

Recent results from the 2-loop frontier

Although there are no complete next-to-next-to-leading (NNLO) predictions for $2 \rightarrow 2$ scattering reactions and $1 \rightarrow 3$ decays (with one truly massive leg) available yet, enormous progress was reached in this direction in recent years.

Complete virtual two-loop amplitudes for (massless) Bhabha scattering [22], light-by-light scattering [23], and $e^+e^- \rightarrow 3$ jets [24] have been worked out, using a large variety of special techniques, which have been summarized in Ref. [25]. A survey of similar results relevant for hadron-collider physics can also be found there. Apart from this two-loop progress on massless particle scattering, also a first step has been made towards massive Bhabha scattering in Ref. [26].

Full NNLO calculations have to include real double-parton bremsstrahlung as well as interference contributions of one-parton bremsstrahlung and one-loop diagrams. The major complication in these parts concerns the proper extraction of the infrared (soft and collinear) singularities. The general form of multiple singular particle emission has been worked out in Ref. [27], which can serve as a basis for the extraction of the singularities. The actual separation of the singularities can be performed either by applying phase-space cuts ("slicing approach") or by subtracting an auxiliary cross section with the same singular structure as the original integrand ("subtraction approach"). In Ref. [28] subtraction terms have been constructed for the leading colour contribution to $e^+e^- \rightarrow 2$ jets in NNLO. However, suitable general subtraction terms as well as their integrated counterparts that have to be added again are not yet available.

Electroweak radiative corrections at high energies

Electroweak corrections far above the electroweak scale, e.g. in the TeV range, are dominated by soft and collinear gauge-boson exchange, leading to corrections of the form $\alpha^N \ln^M(s/M_W^2)$ with $M \leq 2N$. The leading terms ($M = 2N$) are called Sudakov logarithms. At the one-loop ($N = 1$) and two-loop ($N = 2$) level the leading and subleading corrections to a $2 \rightarrow 2$ process at $\sqrt{s} \sim 1\,\text{TeV}$ typically amount to [29]

$$
\begin{aligned}
\delta_{\text{LL}}^{1-\text{loop}} &\sim -\frac{\alpha}{\pi s_w^2} \ln^2\left(\frac{s}{M_W^2}\right) \simeq -26\%, \\
\delta_{\text{NLL}}^{1-\text{loop}} &\sim +\frac{3\alpha}{\pi s_w^2} \ln\left(\frac{s}{M_W^2}\right) \simeq 16\%, \\
\delta_{\text{LL}}^{2-\text{loop}} &\sim +\frac{\alpha^2}{2\pi^2 s_w^4} \ln^4\left(\frac{s}{M_W^2}\right) \simeq 3.5\%, \\
\delta_{\text{NLL}}^{2-\text{loop}} &\sim -\frac{3\alpha^2}{\pi^2 s_w^4} \ln^3\left(\frac{s}{M_W^2}\right) \simeq -4.2\%, \quad (2)
\end{aligned}
$$

revealing that these corrections become significant in the high-energy phase of a future LC. In contrast to QED and QCD, where the Sudakov logarithms cancel in the sum of virtual and real corrections, these terms need not compensate in the electroweak SM for two reasons. The weak charges of quarks, leptons, and electroweak gauge bosons are open, not confined, i.e. there is (in contrast to QCD) no need to average or to sum over gauge multiplets in the initial or final states of processes. Even for final states that are inclusive with respect to the weak charges Sudakov logarithms do not completely cancel owing to the definite weak charges in the initial state [30]. Moreover, the large W- and Z-boson masses make an experimental discrimination of real W- or Z-boson production possible, in contrast to unobservable soft-photon or gluon emission.

In recent years several calculations of these high-energy logarithms have been carried out in the Sudakov regime, where all kinematical invariants $(p_i p_j)$ of different particle momenta p_i, p_j are much larger than all particle masses.[2] A complete analysis of all leading and subleading logarithms at the one-loop level can be found in Ref. [31]. Diagrammatic calculations of the leading two-loop Sudakov logarithms have been carried out in Refs. [29, 32]. Diagrammatic results on the so-called "angular-dependent" subleading logarithms have been presented in Ref. [29]. All these explicit results are compatible with proposed resummations [33, 34] that are based on a symmetric SU(2)×U(1) theory at high energies matched with QED at the electroweak scale. In this ansatz, improved matrix elements $\mathcal{M}$ result from lowest-order matrix elements $\mathcal{M}_0$ upon dressing them with (operator-valued) exponentials,

$$
\mathcal{M} \sim \mathcal{M}_0 \otimes \exp\left(\delta_{\text{ew}}\right) \otimes \exp\left(\delta_{\text{em}}\right). \quad (3)
$$

Explicit expressions for the electroweak and electromagnetic corrections δ_{ew} and δ_{em}, which do not commute with each other, can, for instance, be found in Ref. [29]. For $2 \rightarrow 2$ neutral-current processes of four massless fermions, even subsubleading logarithmic corrections have been derived and resummed [34] using an infrared evolution equation that follows the pattern of QCD.

In supersymmetric models the form of radiative corrections at high energies has also been worked out for a broad class of processes [35]. Based on one-loop results their exponentiation has been proposed.

Higher-order initial-state radiation

Photon radiation off initial-state electrons and positrons leads to large radiative corrections of the form $\alpha^N \ln^N(m_e^2/s)$. These logarithmic corrections are universal and governed by the DGLAP evolution equations. The solution of these equations for the electron-photon system yields so-called structure functions, generically denoted by $\Gamma(x)$ below, which can be used via convolution to improve hard scattering cross sections $\hat{\sigma}(p_{e^+}, p_{e^-})$ by

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[2]Note that this regime does not cover the case of forward scattering of particles, which is also of interest in several cases.

photon emission effects,

$$\sigma(p_{e^+}, p_{e^-}) = \int_0^1 \mathrm{d}x_+ \, \Gamma(x_+) \int_0^1 \mathrm{d}x_- \, \Gamma(x_-)$$
$$\times \, \hat{\sigma}(x_+ p_{e^+}, x_- p_{e^-}). \qquad (4)$$

While the soft-photon part of the structure functions ($x \to 1$) can be resummed, resulting in an exponential form, the contributions of hard photons have to be calculated order by order in perturbation theory. In Ref. [36] the structure functions are summarized up to $\mathcal{O}(\alpha^3)$. Ref. [37] describes a new calculation of the (non-singlet) contributions up to $\mathcal{O}(\alpha^5)$ and of the small-x terms $[\alpha \ln^2(x)]^N$ to all orders (for previous calculations see papers cited in Ref. [37]).

RADIATIVE CORRECTIONS TO
$2 \to 3, 4, \ldots$ PROCESSES

W-pair production and four-fermion final states

The theoretical treatment and the presently gained level in accuracy in the description of W-pair-mediated $4f$ production were triggered by LEP2, as it is reviewed in Refs. [36, 38]. The W bosons are treated as resonances in the full $4f$ processes, $e^+e^- \to 4f \, (+\gamma)$. Radiative corrections are split into universal and non-universal corrections. The former comprise leading-logarithmic corrections from ISR, higher-order corrections included by using appropriate effective couplings, and the Coulomb singularity. These corrections can be combined with the full lowest-order matrix elements easily. The remaining corrections are called non-universal, since they depend on the process under investigation. For LEP2 accuracy, it was sufficient to include these corrections by the leading term of an expansion about the two W poles, defining the so-called double-pole approximation (DPA). Different versions of such a DPA have been used in the literature [39, 40, 41]. Although several Monte Carlo programs exist that include universal corrections, only two event generators, YFSWW [40] and RACOONWW [41, 42], include non-universal corrections.

In the DPA approach, the W-pair cross section can be predicted within $\sim 0.5\%(0.7\%)$ in the energy range between $180 \,\mathrm{GeV}$ ($170 \,\mathrm{GeV}$) and $\sim 500 \,\mathrm{GeV}$, which was sufficient for the LEP2 accuracy of $\sim 1\%$ for energies $170-209 \,\mathrm{GeV}$. At threshold ($\sqrt{s} \lesssim 170 \,\mathrm{GeV}$), the present state-of-the-art prediction results from an improved Born approximation based on leading universal corrections only, because the DPA is not reliable there, and thus possesses an intrinsic uncertainty of about 2%. In Figure 2 this uncertainty is compared with the sensitivity of the W-pair production cross section to the W-boson mass and some assumed experimental data points of a threshold scan. The figure demonstrates the necessary theoretical improvements. At energies beyond $500 \,\mathrm{GeV}$ effects beyond $\mathcal{O}(\alpha)$, such as the above-mentioned Sudakov logarithms at higher orders, become important and have to be included in predictions at per-cent accuracy.
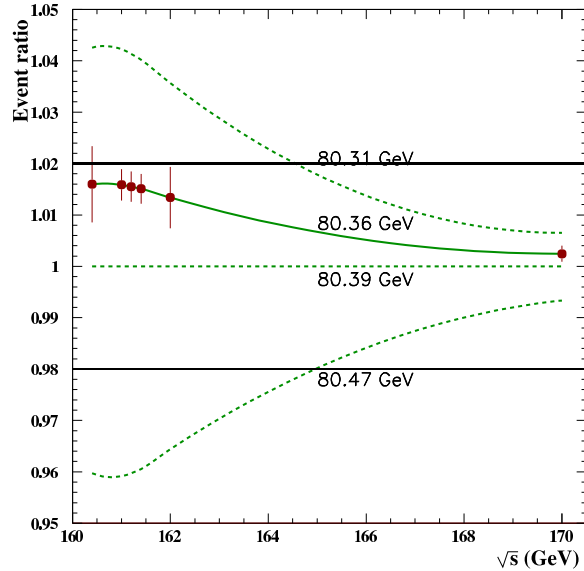
Figure 2: Sensitivity of the W-pair production cross section to the W-boson mass and some assumed experimental data points, compared with the theoretical uncertainty of $\sim 2\%$ (taken from Ref. [1])
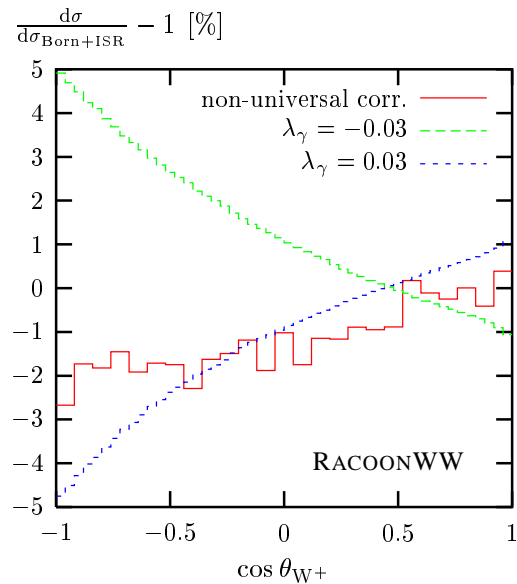
Figure 3: Influence of the anomalous triple gauge-boson coupling λ_γ and of non-universal corrections in the W^+-production-angle distribution at $\sqrt{s} = 200 \,\mathrm{GeV}$ for the process $e^+e^- \to u\bar{d}\mu^-\bar{\nu}_\mu$ (taken from Ref. [45])

At LEP2, the W-boson mass is determined by the reconstruction of the W bosons from their decay products with a final accuracy of about $30\,\mathrm{MeV}$. In Ref. [43] the theoretical uncertainty is estimated to be of the order of $\sim 5\,\mathrm{MeV}$, which is comparable to the estimated [44] accuracy of $\sim 10\,\mathrm{MeV}$ at a future LC. Theoretical improvements are, thus, desirable.

The main sensitivity of all observables to anomalous couplings in the triple-gauge-boson vertices is provided by the W-pair production angle distribution. Figure 3 shows the impact of an anomalous coupling $\lambda_\gamma = \pm 0.03$, the size of which is of the order of the LEP2 sensitivity, together with the impact of non-universal corrections on the spectrum. The theoretical uncertainty in constraining the parameter combination $\lambda = \lambda_\gamma = \lambda_Z$ was estimated to be about 0.005 [46] for the LEP2 analysis. Since a future LC is more sensitive to anomalous couplings than LEP2 by more than an order of magnitude, a further reduction of the uncertainties by missing radiative corrections is necessary. However, a thorough estimate of the theoretical uncertainty in the determination of anomalous couplings at higher scattering energies ($\sqrt{s} \gtrsim 500\,\mathrm{GeV}$), where the experimental sensitivity to non-standard couplings increases, is not yet available.

The above discussion illustrates the necessity of a full one-loop calculation for the $\mathrm{e^+e^-} \to 4f$ process and of further improvements by leading higher-order corrections.

Single-W production

The single-W production process $\mathrm{e^+e^-} \to \mathrm{e}\nu_\mathrm{e}\mathrm{W} \to \mathrm{e}\nu_\mathrm{e} + 2f$ plays a particularly important role among the $4f$ production processes at high scattering energies. The process is predominantly initiated by $\mathrm{e}\gamma^*$ collision (see Figure 4) where the photon is radiated off the electron (or positron) by the Weizsäcker–Williams mechanism, i.e. with a very small off-shellness q_γ^2. Consequently the cross section rises logarithmically with the scattering energy and is of the same size as the W-pair production cross section at about $\sqrt{s} = 500\,\mathrm{GeV}$; for higher energies single-W dominates over W-pair production.

Theoretically the dominance of photon exchange at low q_γ^2 poses several complications. Technically, $q_\gamma^2 \to 0$ means that the electrons (or positrons) are produced in the forward direction and that the electron mass has to be taken

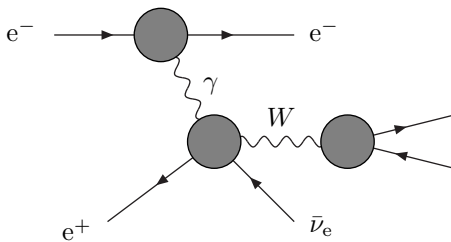Figure 4: Generic diagram for the dominant contributions to single-W production, $\mathrm{e^+e^-} \to \mathrm{e^-}\nu_\mathrm{e}\mathrm{W} \to \mathrm{e^-}\nu_\mathrm{e} + 2f$

into account in order to describe the cross section there. Moreover, the mere application of s-dependent leading-logarithmic structure functions does not describe the leading photon-radiation effects properly, since ISR and final-state radiation (FSR) show sizeable interferences for forward scattering. Thus, the improvement of lowest-order calculations by leading radiation effects is more complicated than for s-channel-like processes. Finally, the running of the electromagnetic coupling $\alpha(q_\gamma^2)$ has to be evaluated in the region of small momentum transfer ($q_\gamma^2 < 0$) where the fit [47] of this quantity to the hadronic vacuum polarisation should be used.

The Monte Carlo generator KORALW [48] has recently been updated to include the ISR-FSR interference effects as well as the proper running of $\alpha(q_\gamma^2)$. Therefore, this program now has reached a level of accuracy similar to the other state-of-the-art programs for single-W production: GRC4F [49], NEXTCALIBUR [50], SWAP [51], WPHACT [52], and WTO [53]. More detailed descriptions of these codes can be found in Ref. [38]. It should be kept in mind that none of these calculations includes non-universal electroweak corrections, leading to a theoretical uncertainty of about $\sim 5\%$ in cross-section predictions. Although the final solution for a high-energy LC certainly requires a full $\mathcal{O}(\alpha)$ calculation of the $4f$-production process, a first step of improvement could be done by a careful expansion about the propagator poles of the photon and W boson. The electroweak $\mathcal{O}(\alpha)$ corrections to the process $\mathrm{e}\gamma \to \nu_\mathrm{e}\mathrm{W}$, which are known [54], represent a basic building block in this calculation.

Technical progress on radiative corrections to multi-particle production processes

One-loop integrals become more and more cumbersome if the number N of external legs in diagrams increases. For $N > 4$, however, not all external momenta are linearly independent because of the four-dimensionality of space-time. As known for a long time [55], this fact opens the possibility to relate integrals with $N > 4$ to integrals with $N \leq 4$. In recent years, various techniques for actual evaluations of one-loop integrals with $N = 5, 6$ have been worked out [56, 57] (see also references therein for older methods and results). The major complication in the treatment of $2 \to 3$ processes at one loop concerns the numerical evaluation of tensor 5-point integrals; in particular, the occurrence of inverse Gram determinants in the usual Passarino–Veltman reduction to scalar integrals leads to numerical instabilities at the phase-space boundary. A possible solution to this problem was worked out in Ref. [57] where the known direct reduction [55] of scalar 5-point to 4-point integrals was generalized to tensor integrals, thereby avoiding the occurrence of leading Gram determinants completely.

In the evaluation of real corrections, such as bremsstrahlung, a proper and numerically stable separation of infrared (soft and collinear) divergences represents one of

the main problems. In the phase-space slicing approach (see Ref. [58] and references therein) already mentioned above, the singular regions are excluded from the "regular" phase-space integration by small cuts on energies, angles, or invariant masses. Using factorization properties, the integration over the singular regions can be done in the limit of infinitesimally small cut parameters. The necessary fine-tuning of cut parameters is avoided in so-called subtraction methods (see Refs. [59, 60, 61] and references therein), where a specially tuned auxiliary function is subtracted from the singular integrand in such a way that the resulting integral is regular. The auxiliary function has to be chosen simple enough, so that the singular regions can be integrated over analytically. In Ref. [59] the so-called "dipole subtraction approach" has been worked out for massless QCD. The technique admits a convenient construction of such auxiliary functions for arbitrary one-parton emission processes, without the need of any further complicated analytical integrations. The dipole subtraction formalism was subsequently worked out for photon emission off massive fermions in Ref. [60] and for QCD with massive quarks in Ref. [61].

Results on $2 \to 3$ processes at one-loop order – $e^+e^- \to \nu\bar{\nu}H, t\bar{t}H$

Recently some one-loop calculations of electroweak radiative corrections have been presented for $2 \to 3$ processes that are interesting at a future LC: $e^+e^- \to \nu\bar{\nu}H$ [63, 64] and $e^+e^- \to t\bar{t}H$ [65, 66, 67]. The results of Refs. [63, 66] were obtained with the GRACE-LOOP [68] system (see below). In Refs. [64, 65, 67] the technique [57] for treating tensor 5-point integrals was employed. While Refs. [63, 65, 66] make use of the slicing approach for treating soft-photon emission, the results of Refs. [64, 67] have been obtained by dipole subtraction and checked by phase-space slicing for soft and collinear bremsstrahlung.

In e^+e^- annihilation there are two main production mechanisms for the SM Higgs boson. In the Higgs-strahlung process, $e^+e^- \to ZH$, a virtual Z boson decays into a Z boson and a Higgs boson. The corresponding cross section rises sharply at threshold ($\sqrt{s} \gtrsim M_Z + M_H$) to a maximum at a few tens of GeV above $M_Z + M_H$ and then falls off as s^{-1}, where $\sqrt{s}$ is the CM energy of the e^+e^- system. In the W-boson-fusion process, $e^+e^- \to \nu_e\bar{\nu}_eH$, the incoming e^+ and e^- each emit a virtual W boson which fuse into a Higgs boson. The cross section of the W-boson-fusion process grows as $\ln s$ and thus is the dominant production mechanism for $\sqrt{s} \gg M_H$. At the one-loop level, first the contributions of fermion and sfermion loops in the Minimal Supersymmetric Standard Model (MSSM) have been evaluated in Ref. [62]. A complete calculation of the $\mathcal{O}(\alpha)$ electroweak corrections to $e^+e^- \to \nu\bar{\nu}H$ in the SM has subsequently been performed in Refs. [63, 64].[3] Some

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[3]Analytical results for the one-loop corrections have been obtained in Ref. [69] as MAPLE output, but a numerical evaluation of these results is not yet available.

Table 1: Comparison of lowest-order cross sections for $e^+e^- \to \nu\bar{\nu}H$, (σ_{tree}), of one-loop-corrected cross section (σ), and of relative corrections ($\delta = \sigma/\sigma_{\mathrm{tree}} - 1$) between Refs. [63, 64] at $\sqrt{s} = 500\,\mathrm{GeV}$ (input parameters of Ref. [63])

$M_H[\mathrm{GeV}]$	$\sigma_{\mathrm{tree}}[\mathrm{fb}]$	$\sigma[\mathrm{fb}]$	$\delta[\%]$	Ref.
150	61.074(7)	60.99(7)	-0.2	[63]
	61.076(5)	60.80(2)	$-0.44(3)$	[64]
200	37.294(4)	37.16(4)	-0.4	[63]
	37.293(3)	37.09(2)	$-0.56(4)$	[64]
250	21.135(2)	20.63(2)	-2.5	[63]
	21.134(1)	20.60(1)	$-2.53(3)$	[64]
300	10.758(1)	10.30(1)	-4.2	[63]
	10.7552(7)	10.282(4)	$-4.40(3)$	[64]

Table 2: Comparison of lowest-order cross sections for $e^+e^- \to t\bar{t}H$, (σ_{tree}), of one-loop-corrected cross section (σ), and of relative corrections ($\delta = \sigma/\sigma_{\mathrm{tree}} - 1$) between Refs. [66, 67] for $M_H = 120\,\mathrm{GeV}$ (input parameters of Ref. [66], results taken from Ref. [67])

$\sqrt{s}[\mathrm{GeV}]$	$\sigma_{\mathrm{tree}}[\mathrm{fb}]$	$\sigma[\mathrm{fb}]$	$\delta[\%]$	Ref.
600	1.7293(3)	1.738(2)	0.5	[66]
	1.7292(2)	1.7368(6)	0.44(3)	[67]
800	2.2724(5)	2.362(4)	3.9	[66]
	2.2723(3)	2.3599(6)	3.86(2)	[67]
1000	1.9273(5)	2.027(4)	5.2	[66]
	1.9271(3)	2.0252(5)	5.09(2)	[67]

results of Refs. [63, 64] are compared in Table 1. The agreement of the correction is within 0.2% or better w.r.t. the lowest-order cross sections.

The Yukawa coupling of the top quark could be measured at a future LC with high energy and luminosity at the level of $\sim 5\%$ [1] by analyzing the process $e^+e^- \to t\bar{t}H$. A thorough prediction for this process, thus, has to control QCD and electroweak corrections. Table 2 summarizes some results on the electroweak $\mathcal{O}(\alpha)$ corrections of Refs. [66, 67]. The agreement within $\sim 0.1\%$ also holds for other energies and Higgs-boson masses. The results of the previous calculation [65] roughly agree with the ones of Refs. [66, 67] at intermediate values of $\sqrt{s}$ and M_H, but are at variance at high energies (TeV range) and close to threshold (large M_H).

EVENT GENERATORS FOR MULTI-PARTICLE FINAL STATES

Multi-purpose generators at parton level

The large variety of different final states for multi-particle production renders multi-purpose Monte Carlo event generators rather important, i.e. generators that deliver an event generator for a user-specified (as much as possible) general final state based on full lowest-order amplitudes. As results, these tools yield lowest-order predictions for observables, or more generally Monte Carlos samples of events, that are improved by universal radiative corrections, such as initial-state radiation at the leading-logarithmic level or beamstrahlung effects. Most of the multi-purpose generators are also interfaced to parton-shower and hadronization programs. The generality renders these programs, however, rather complex devices and, at present, they are far from representing tools for high-precision physics, because non-universal radiative corrections are not taken into account in predictions.

The following multi-purpose generators for multi-parton production, including program packages for the matrix-element evaluation, are available:

- AMEGIC [70]: Helicity amplitudes are automatically generated by the program for the SM, the MSSM, and some new-physics models. Various interfaces (ISR, PDFs, beam spectra, ISAJET, etc.) are supported. The phase-space generation was successfully tested for up to six particles in the final state.

- GRACE [71]: The amplitudes are delivered by a built-in package, which can also handle SUSY processes. The phase-space integration is done by BASES [72]. Tree-level calculations have been performed for up to (selected) six-fermion final states. The extension of the system to include one-loop corrections, the GRACE-LOOP [68] program, is under construction.

- MADEVENT [73] + MADGRAPH [74]: The MAD-GRAPH algorithm can generate tree-level matrix elements for any SM process (fully supporting particle masses), but a practical limitation is 9,999 diagrams. In addition, MADGRAPH creates MADEVENT, an event generator for the requested process.

- PHEGAS [75] + HELAC [76]: The HELAC program delivers amplitudes for all SM processes (including all masses). The phase-space integration done by PHEGAS has been tested for selected final states with up to seven particles.

- WHIZARD [77] + COMPHEP [78] / MADGRAPH [74] / O'MEGA [79]: Matrix elements are generated by an automatic interface to (older versions of) COMPHEP, MADGRAPH, and (the up-to-date version of) O'MEGA. Phase-space generation has been tested for most $2 \to 6$ and some $2 \to 8$ processes; unweighed events are supported, and a large variety of interfaces (ISR, beamstrahlung, PYTHIA, PDFs, etc.) exists. The inclusion of MSSM amplitudes (O'MEGA) and improved phase-space generation ($2 \to 6$) are work in progress.

All but the GRACE program make use of the multi-channel approach for the phase-space integration. More details can be found in the original references.

Tuned comparisons of different generators, both at parton and detector level, are extremely important, but become more and more laborious owing to the large variety of multi-particle final states. Some progress to a facilitation and automization of comparisons are made by MC-tester project [80] and Java interfaces [81].

Event generators and results for $e^+e^- \to 6f$

Particular progress was reached in recent years in the description of six-fermion production processes. Apart from the multi-purpose generators listed in the previous section, also dedicated Monte Carlo programs and generators have been developed for this class of processes:

- SIXFAP [82]: Matrix elements are provided for all $6f$ final states (with finite fermion masses), including all electroweak diagrams. The generalization to QCD diagrams and the extension of the phase-space integration for all final states is in progress.

- EETT6F [83]: Only processes relevant for $t\bar{t}$ production are supported (a new version includes $e^{\pm}$ in the final state and QCD diagrams); finite fermion masses are possible.

- LUSIFER [84]: All $6f$ final states are possible, including QCD diagrams with up to four quarks; representative results for all these final states have been presented. External fermions are massless. An unweighting algorithm and an interface to PYTHIA are available.

Table 3 summarizes a brief comparison of results for some processes $e^+e^- \to 6f$ relevant for $t\bar{t}$ production for massless external fermions. The results reveal good agreement between the various programs, where minor differences are presumably due to the different treatments of the bottom-quark Yukawa coupling, which is neglected in some cases.

A tuned comparison of results obtained with LUSIFER and WHIZARD for a large survey of $6f$ final states has been presented in Ref. [84].

"True" Monte Carlo event generators

The event generators described above work at parton level (partially improved by parton showers), i.e. the final-state particles cannot be directly identified with particles in a detector. For detector simulations, these parton-level generators have to be interfaced with parton shower and hadronization programs. To facilitate this interface, the

Table 3: Comparison of lowest-order predictions for some processes $e^+e^- \to t\bar{t} \to 6$ fermions at $\sqrt{s} = 500\,\text{GeV}$ in approximation of massless fermions (input parameters and cuts of Ref. [84])

$\sigma_{\text{full}}[\,\text{fb}]$	AMEGIC++	EETT6F	LUSIFER	PHEGAS	SIXFAP	WHIZARD
$\nu_e e^+ e^- \bar{\nu}_e b\bar{b}$	5.879(8)	5.862(6)	5.853(7)	5.866(9)	5.854(3)	5.875(3)
$\nu_e e^+ \mu^- \bar{\nu}_\mu b\bar{b}$	5.827(4)	5.815(5)	5.819(5)	5.822(7)	5.815(2)	5.827(3)
$\nu_\mu \mu^+ \mu^- \bar{\nu}_\mu b\bar{b}$	5.809(5)	5.807(3)	5.809(5)	5.809(5)	5.804(2)	5.810(3)
$\nu_\mu \mu^+ \tau^- \bar{\nu}_\tau b\bar{b}$	5.800(3)	5.797(5)	5.800(4)	5.798(4)	5.798(2)	5.796(3)
$\nu_\mu \mu^+ d\bar{u} b\bar{b}$	17.209(9)	17.213(23)	17.171(24)	17.204(18)		
without QCD:	17.097(8)	17.106(15)	17.095(11)	17.107(18)	17.096(4)	17.103(8)

"Les Houches accord" [85] has been designed, a set of FORTRAN common blocks for the transfer of event configurations from parton level generators to showering and hadronization event generators. Alternatively so-called "true" event generators could be used, which fully include hadronization. The following well-known "true" MC generators represent general-purpose tools for investigating not only e^+e^- collisions, but also lepton–hadron and hadron–hadron scattering: HERWIG [86], ISAJET [87], and PYTHIA [88]. The programs are supported and extended continuously; recent upgrades and new features relevant for LC physics are:

- implementation of all $2 \to 2$ scattering processes of the MSSM in lowest order;

- associated Higgs production ($Q\bar{Q}^{(\prime)}H$) in the SM and MSSM in lowest order (the case of charged Higgs bosons is not yet available in ISAJET and PYTHIA);

- R-parity-violation SUSY in HERWIG;

- MSSM with complex parameters (cMSSM) in PYTHIA;

- inclusion of spin correlations and matrix elements for 3- and 4-body decays in HERWIG;

- introduction of real corrections based on matrix elements for several e^+e^- processes in HERWIG and PYTHIA (see e.g. Ref. [89]).

Among other work in progress, the implementation of NLO QCD corrections in "true" generators is one of the most pressing issues. In particular, the matching of parton showers with matrix-element calculations at NLO has to be performed carefully; first results look very promising [90].

Finally, it should be mentioned that the present FORTRAN versions of HERWIG and PYTHIA will be replaced by C++ programs in the future [91].

RADIATIVE CORRECTIONS IN SUPERSYMMETRIC THEORIES

In order to avoid too much overlap with the reports of the Higgs and SUSY groups, this section is mainly restricted
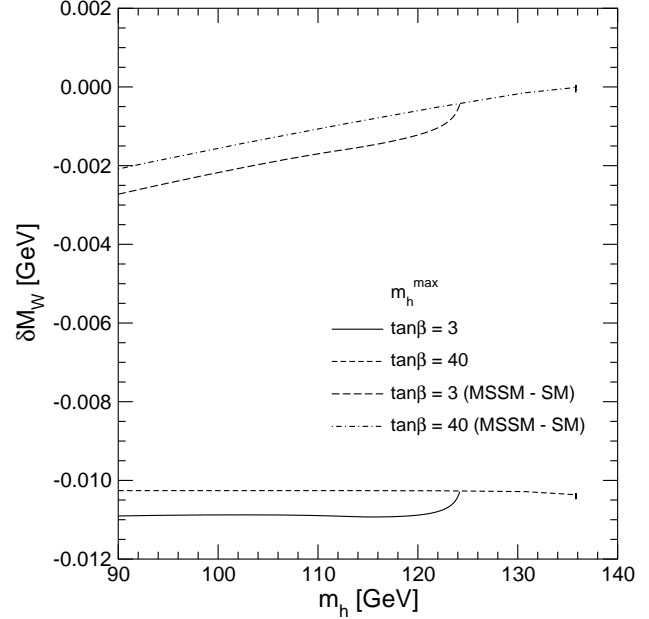

Figure 5: Effect of the SUSY $\mathcal{O}(\alpha_t^2)$ corrections on the prediction of M_W in the MSSM (taken from Ref. [93])

to the topics that have been presented in the Loopverein working group. More details and references on the subject can be found in Ref. [5].

SUSY corrections to precision observables

The confrontation of high-precision data with theoretical predictions is, of course, also very interesting in extensions of the SM. The one-loop corrections of the MSSM to muon decay and to the pseudo-observables of the Z resonance have been known for many years, but not many corrections beyond one loop exist. Recently the known universal corrections of $\mathcal{O}(\alpha\alpha_s)$ [92] to the ρ-parameter have been supplemented by the terms of $\mathcal{O}(\alpha_t^2)$, $\mathcal{O}(\alpha_t\alpha_b)$ and $\mathcal{O}(\alpha_b^2)$ in Ref. [93], where $\alpha_{t/b} = h_{t/b}^2/(4\pi)$ with $h_{t/b}$ denoting the top/bottom Yukawa coupling. Figure 5 illustrates the effect of the $\mathcal{O}(\alpha_t^2)$ corrections on the prediction of the W-boson mass in the MSSM. The genuine MSSM $\mathcal{O}(\alpha_t^2)$ effects modify M_W at the level of $2-3\,\text{MeV}$.

Mass spectra in the MSSM

In theories with unbroken supersymmetry the fermions and bosons within the same multiplet have a common mass. In realistic theories, such as the MSSM, SUSY is broken, and this statement is not valid anymore. However, the masses of fermions or bosons within multiplets are not all independent, i.e. there are non-trivial relations among mass parameters. Since the mass spectra of SUSY theories bear a lot of information on the mechanism of SUSY breaking, precision analyses of these spectra can serve as a window to grand unification.

SUSY demands (at least) two Higgs doublets to give the up- and down-type fermions masses. Thus, the MSSM predicts the existence of two charged ($H^{\pm}$), two neutral scalar (h^0, H^0), and one neutral pseudo-scalar (A^0) Higgs bosons. In lowest order, the Higgs masses $M_{H^{\pm}}$, $M_{\rm h}$, and $M_{\rm H}$ can be calculated as functions of the A^0-boson mass M_A, the ratio of Higgs-field vacuum expectation values, $\tan\beta = v_2/v_1$, and the gauge-boson masses; in particular, the mass $M_{\rm h}$ of the lightest Higgs boson is constrained to be smaller than M_Z at tree level. Beyond lowest order, also the remaining MSSM parameters are involved in the mass relations, and $M_{\rm h}$ can reach values up to about $135\,{\rm GeV}$. The status of precision calculations of the neutral Higgs-boson masses has been recently reviewed in Ref. [94]. All available corrections[4] are implemented in the program FEYNHIGGS [96]. The predictions are based on full one-loop calculations and on the leading effects in two-loop order, i.e. the corrections of the order $\mathcal{O}(\alpha_s\alpha_t)$, $\mathcal{O}(\alpha_t^2)$ and $\mathcal{O}(\alpha_s\alpha_b)$ (see Ref. [94] and references therein). Recently the corrections of $\mathcal{O}(\alpha_t\alpha_b)$ and $\mathcal{O}(\alpha_b^2)$ have become available [97]. The current theoretical uncertainty in the Higgs-mass predictions is about $3\,{\rm GeV}$ [94], but a further reduction to $\lesssim 0.5\,{\rm GeV}$ should be reached to match the accuracy needed for a LC. In this context a proper definition of $\tan\beta$ in higher perturbative orders is crucial, since different renormalization schemes (see, e.g., Ref. [98]) for $\tan\beta$ lead to rather different relations between $\tan\beta$ and physical observables such as the Higgs-boson masses.

In the sector of charginos and neutralinos of the MSSM only three out of the six mass parameters are independent. In Refs. [99, 100] the three masses $m_{\tilde{\chi}^0_{2,3,4}}$ of the heavier neutralinos have been expressed in terms of the mass $m_{\tilde{\chi}^0_1}$ of the lightest neutralino and of the masses $m_{\tilde{\chi}^+_{1,2}}$ of the two charginos, including the complete one-loop corrections, which depend also on the other MSSM parameters. The corrections modify the calculated masses by up to several GeV. The on-shell renormalization in the chargino-neutralino sector is described in Refs. [99, 100, 101] in detail.

The relations among sfermion masses, together with the

corresponding on-shell renormalization, are worked out in Refs. [101, 102]. For each generation, one of the four squark masses and one of the three slepton masses can be calculated from the other sfermion masses (and the other MSSM parameters entering at one loop). The one-loop corrections can amount to about 5%.

Higgs-boson and SUSY-particle decays in the MSSM

Analyses of particle decays are of great importance for the reconstruction of coupling structures and, thus, of interaction Lagrangians. The rich particle content of SUSY theories leads to a large variety of decay cascades, which depend in detail on the chosen scenario. A discussion of phenomenological implications and of radiative corrections to SUSY-particle decays can be found in Ref. [5] and Ref. [103], respectively.

The decay widths of Higgs bosons in the MSSM received much attention in recent years, so that the predictions are well elaborate. Precise predictions can be obtained with the programs FEYNHIGGSDECAY (based on Ref. [104]) and HDECAY [105]. Recently the electroweak $\mathcal{O}(\alpha)$ corrections to the decay of the CP-odd A^0 boson into sfermion pairs have been calculated in Ref. [106]. For the Higgs decay $\phi \to {\rm b\bar{b}}$ ($\phi = {\rm h}^0, {\rm H}^0, {\rm A}^0$), which is of particular importance for light Higgs bosons, the resummation of the leading SUSY-QCD effects and the related theoretical uncertainty have been discussed in Ref. [107] (for previous work on $\phi \to {\rm b\bar{b}}$ see references therein).

Apart from considering integrated decay rates, it is interesting to inspect distributions of decay products, which is important for the determination of the spin and parity of the decaying particle. This task requires the development of appropriate Monte Carlo tools. For the $\phi \to \tau^+\tau^-$ decay, for instance, the TAUOLA program was extended to include τ-spin correlations in Ref. [108].

SUSY-particle production

The direct production of SUSY particles, if they exist, is among the most interesting issues at future colliders. In order to determine the properties (mass, spin, decay widths, couplings) of these new particles, precise measurements and predictions of the corresponding cross sections at the same level of accuracy are necessary, i.e. radiative corrections have to be taken into account.

The electroweak radiative corrections to the production of sfermions, ${\rm e^+e^-} \to \tilde{f}\bar{\tilde{f}}$, and charginos, ${\rm e^+e^-} \to \tilde{\chi}^+\tilde{\chi}^-$, were worked out in Refs. [109] and [110], respectively. Since in these calculations the sfermions and charginos are assumed to be stable, the results are relevant for energies a few decay widths above the production threshold. For a threshold scan the decay of the sfermions as well as non-resonant coherent background effects have to be included; in Ref. [111] off-shell effects in sfermion-pair production have been investigated at tree level (improved

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[4]It should be mentioned that the full effective potential has been presented in Ref. [95] at the two-loop level. However, the precise relation of these results with the parameters defined in the on-shell renormalization scheme used in MSSM parameter analyses has not been worked out so far.

by universal Coulomb-like corrections). Theoretically the whole issue is very similar to a description of the process $e^+e^- \to WW \to 4f$ which is discussed above.

The SUSY multiplet structure, in particular, predicts that the strengths of the gauge-boson–fermion and gauge-boson–sfermion interactions, which are equal owing to the gauge principle, coincide with the gaugino–sfermion–fermion Yukawa coupling. In order to test this relation in SUSY QCD the processes $e^+e^- \to q\bar{q}g, \tilde{q}\bar{\tilde{q}}g, \tilde{q}\bar{q}\tilde{g}, q\bar{\tilde{q}}\tilde{g}$ should be studied. In Ref. [112] the QCD and SUSY-QCD corrections to these processes were calculated.

More details on radiative corrections to SUSY particle production and decays can be found in Ref. [103] (and references therein).

Renormalization of the MSSM beyond one loop

Beyond one loop the calculation of radiative corrections within SUSY theories is highly non-trivial, because there is no regularization scheme that respects gauge invariance and supersymmetry at the same time. For instance, conventional dimensional regularization breaks supersymmetry, while dimensional reduction, which respects supersymmetry, is known to be not fully consistent. In this situation, a mathematically convincing way to perform renormalization is provided by *algebraic renormalization*. In this framework the symmetry identities and a proof of renormalizability for the MSSM have been established in Ref. [113]. These results can serve as a basis for the construction of all symmetry-restoring counterterms in the MSSM.

OTHER DEVELOPMENTS

Automization of loop calculations and maintenance of computer codes

Once the necessary techniques and theoretical subtleties of a perturbative calculation are settled, to carry out the actual calculation is an algorithmic matter. Thus, an automization of such calculations is highly desirable, in order to facilitate related calculations. Various program packages have been presented in the literature for automatized tree-level, one-loop, and multi-loop calculations. A comprehensive overview was, for instance, given in Ref. [114]; in the following we have to restrict ourselves to a selection of topics, where the emphasis is put on recent developments.

The generation of Feynman graphs and amplitudes is a combinatorial problem that can be attacked with computer algebra. The program packages FEYNARTS [7] (which has been extended [115] for the MSSM), QGRAF [14], DIANA [11] (based on QGRAF) are specifically designed for this task; also the GRACE-LOOP [68] system automatically generates Feynman diagrams and loop amplitudes. Moreover, the task of calculating virtual one-loop and the corresponding real-emission corrections to $2 \to 2$ scattering reactions is by now well understood. Such calculations are widely automated in the packages FORMCALC, combined with LOOPTOOLS [116], and GRACE-LOOP [68].

As an illustrating example, Table 4 provides some results on the differential cross section for $e^+e^- \to t\bar{t}$ in lowest order as well as including electroweak $\mathcal{O}(\alpha)$ corrections. The program FA+FC was obtained from the output of the FEYNARTS and FORMCALC packages and makes use of the LOOPTOOLS library for the numerical evaluation. The TOPFIT program [117, 118] was developed from an algebraic reduction of Feynman graphs (delivered from DIANA) within FORM; for the numerics LOOPTOOLS is partially employed. More detailed comparisons between FA+FC and TOPFIT, including also other fermion flavours, can be found in Refs. [117, 119]. The GRACE-LOOP result is completely independent of the two others. The agreement between these results reflects the enormous progress achieved in recent years in the automization of one-loop calculations.

The GRACE-LOOP system has recently been used in the calculation of the electroweak corrections to the $2 \to 3$ processes $e^+e^- \to \nu\bar{\nu}H, t\bar{t}H$ [63, 66], which are discussed above.

Clearly the calculation of radiative corrections is a very laborious task, leading to rather complex and lengthy computer codes, which should be carefully documented. The SANC project [120] (former CALCPHEP [121]) aims at providing theoretical support of this kind for future accelerator experiments, using the principle of knowledge storing. This approach is rather different from the strategy of automization described above, which aims at generating completely new programs. The SANC program contains another independent calculation of the $\mathcal{O}(\alpha)$ corrections to $e^+e^- \to t\bar{t}$, the results of which are also included in Table 4.

Numerical approaches to loop calculations

Most of the various techniques of performing loop calculations share the common feature that the integration over the loop momenta is performed analytically. This procedure leads to complications at one loop if five or more external legs are involved, since both speed and stability of programs become more and more jeopardized. At the two-loop level, already the evaluation of self-energy and vertex corrections can lead to extremely complicated higher transcendental functions that are hard to evaluate numerically.

An idea to avoid these complications is provided by a more or less purely numerical evaluation of loop corrections. There are two main difficulties in this approach. Firstly, the appearing ultraviolet and infrared divergences have to be treated and canceled carefully. Secondly, even finite loop integrals require a singularity handling of the integrand near so-called particle poles, where Feynman's $i\epsilon$ prescription is used as regularization.

In Ref. [122] a method for a purely numerical evaluation of loop integrals is proposed. Each integral is parametrized with Feynman parameters and subsequently rewritten with

Table 4: Differential cross sections for $e^+e^- \to t\bar{t}$ for selected scattering angles at $\sqrt{s} = 500\,\text{GeV}$; input parameters are defined in Ref. [117], the soft-photon cut parameter $\omega/\sqrt{s}$ is set to 10^{-5}.

$\cos\theta$	program	$\left(\dfrac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\right)_{\text{Born}}$ [pb]	$\left(\dfrac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\right)_{\text{Born+virt+soft}}$ [pb]	$\left(\dfrac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\right)_{\text{Born+virt+real}}$ [pb]
−0.9	FA + FC	0.108839194076039	−0.00205485893415	
	GRACE-LOOP	0.108839194076	−0.002054859	0.13206(12)
	SANC	0.10883919407522	−0.00205485893360	
	TOPFIT	0.108839194076039	−0.00205485893466	0.13229
0.0	FA+FC	0.225470464033559	−0.04321416793299	
	GRACE-LOOP	0.225470464033	−0.043214168	0.23513(14)
	SANC	0.22547046403258	−0.04321416793300	
	TOPFIT	0.225470464033559	−0.04321416793192	0.23476
+0.9	FA+FC	0.491143715767761	−0.16747885864057	
	GRACE-LOOP	0.491143715767	−0.16747886	0.47709(21)
	SANC	0.49114371576694	−0.16747885864510	
	TOPFIT	0.491143715767761	−0.16747885863793	0.47768

partial integrations. The final expression consists of a quite simple part containing the singular terms and another more complicated looking part that can be integrated numerically. The actual application of the method to a physical process is still work in progress.

Another idea was proposed in Ref. [123] and applied to event-shape variables in $e^+e^- \to 3\,\text{jets}$ in NLO. In this approach virtual and real corrections are added before integration. In their sum, no soft singularities, or more generally singularities that cancel between virtual and real corrections, appear from the beginning. Nevertheless the problem of a stable treatment of particle poles in loop amplitudes still remains. In Ref. [123] a solution via contour deformations in complex integration domains is described, but how this procedure can be generalized is not yet clear.

CONCLUSIONS

In spite of the complexity of higher-order calculations for high-energy elementary particle reactions, there has been continuous progress in the development of new techniques and in making precise predictions for physics at future colliders. However, to be prepared for a future e^+e^- linear collider with high energy and luminosity, such as TESLA, an enormous amount of work is still ahead of us. Full two-loop predictions for $e^+e^- \to 2\,\text{fermion}$ scattering reactions, such as the Bhabha process, or full one-loop calculations for $e^+e^- \to 4\,\text{fermions}$ are more than technical challenges. At this level of accuracy, field-theoretical issues such as renormalization, the treatment of unstable particles, etc., demand a higher level of understanding. Of course, both loop calculations as well as the descriptions of multi-particle production processes with Monte Carlo techniques require and will profit from further improving computing devices.

It is certainly out of question that the list of challenges and interesting issues could be continued at will. The way to a future LC will also be highly exciting in precision physics.

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